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Funnel Dynamic Surface Asymptotic Control for Prescribed Performance of Nonlinear Systems

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ABSTRACT : *In this paper, a funnel dynamic surface control (FDSC) scheme is proposed for a class of uncertain nonlinear systems. A new funnel variable is used in the controller design, which achieves prescribed tracking error performance and avoids the potential singularity problem of prescribed performance controls (PPC). Moreover, the transient and steady performance can be significantly improved. By introducing the compensation terms of the boundary layer errors at the recursive steps of dynamic surface control (DSC), the "explosion of complexity" problem is eliminated and the design procedure is simplified. The key advantage of the compensation terms is that the asymptotic tracking with zero error can be held and only one tuning parameter of compensation terms is needed, which greatly reduces the computational burden and makes it easy to implement. Finally, comparative simulations and experimental results have shown that the proposed scheme achieves better tracking performance.*
Keywords: *funnel error variable, dynamic surface control, asymptotic tracking, nonlinear systems*

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I. INTRODUCTION

In the past decades, much research effort has been concentrated on the problem of controlling nonlinear uncertain systems. Significant control developments have been achieved, including adaptive control, robust control, sliding mode control, etc. Especially, the adaptive backstepping approach has been applied as a powerful and effective design method for a large class of nonlinear systems, which are particularly useful for systems in the strict-feedback form. Nevertheless, as many authors have pointed out in [1, 2], a drawback in the traditional backstepping design is the issue of "explosion of complexity" in the step-by-step design procedure. That is, the virtual status and virtual controls are introduced into the backstepping technique [3]. And then the nonlinear functions, virtual controls, should be differentiated repeatedly, which implies that the complexity of controller design procedure grows drastically with the increase of the system order [4]. It inevitably leads to a complicated controller with heavy computation costs and limits its practical implement.

To deal with the issue of the backstepping-based approach, Swaroop et al. [5] proposed a new recursive approach named DSC for a class of strict-feedback nonlinear systems. This issue is eliminated by introducing low pass filters at the traditional backstepping scheme, which makes the virtual control laws passed through the filters at each design step to replace the differentiation operator. What is more, an extra advantage is that it significantly relaxes the requirement that the derivatives of the system functions and reference signal up to a certain order. In [2], this technique is extended to the adaptive control approach, namely the adaptive dynamic surface control. Consequently, a large number of applications have been put into effect by the DSC technique to design low complexity controllers [6–8]. However, as revealed by [9], these adaptive DSC systems with linear low pass filters suffer from large tracking-error bounds, which implies that the tracking performance of DSC approach highly depends on the filter time constants. To address this problem and improve the control performance, the command filter [10], Levant's differentiator [11], the tracking differentiator [12], the robust second-order filters [13] were developed, where these filters were employed to replace the first-order filters in the DSC design. Besides, the influence of boundary layer errors on control systems, which would degrade tracking accuracy, is still not considered. On the other hand, in above DSC control works, they only achieve the bounded-error trajectory tracking, that is the tracking error can be confined to some small residual sets, rather than the asymptotic tracking with zero error. Moreover, the size of the residual sets is often unknown. Also, it is worthy to point out that, until now, a few adaptive DSC control can guarantee the transient as well as the steady-state performance. Therefore, it is still a challenging and valuable work to investigate the adaptive DSC control strategy for uncertain nonlinear systems.

Recently, Bechlioulis and Rovithakis [14] developed a new noticeable prescribed performance control (PPC) for a class of feedback linearisable nonlinear systems to guarantee a expected tracking performance. The key idea of this methodology is that the prescribed performance function (PPF) is provided to transform the tracking error of an original system into a new error system. A tracking error of the transient property and steady-state performance are characterized by PPF. Then, the new transformed error is incorporated into the following controller design. This approach achieves the convergence rate of tracking error no less than a prescribed value and the maximum overshoot less than a sufficiently small constant [15]. And then this technique was successfully extended to strict feedback nonlinear systems by [16]. Since then, several applications and improvements based on this method have been developed. Nevertheless, as S. I. Han et al. pointed out in [17], the PPC method may suffer from a potential singularity problem since the inverse transformation function includes certain constraint conditions. This may result in violation of the prescribed performance constraints and even the instability of the control system [18]. On the contrary to PPC, funnel control (FC), which is developed by Ilchman et al. [19, 20], is a “proportional” (memoryless) method to guarantee a prescribed transient performance and asymptotic tracking of the control systems. Therefore, the complexity of identification and the estimation of systems can be bypassed and the measurement noises and parameter uncertainties can be tolerated. Then, Hackl et al. successfully extended this technique to more generic systems [21, 22]. However, the application of FC is limited to a class of systems *S* with relative degree one or two [23], thus, in [24], S. I. Han et al. proposed a new transformation function to avoid this obstacle and made both theoretical studies and practical application more convenient.

Inspired by previous researches, this paper will present a new funnel dynamic surface control (FDSC) scheme to guarantee the transient behavior and achieve the asymptotic output tracking for uncertain nonlinear strict feedback systems. Compared to the relevant existing works, the main contributions of this paper are summarized below.

i. By employing the output tracking error in the funnel constraint function, FC method is adopted in the DSC procedure. Compared with current PPC control methods, the potential singularity problem is avoided. Moreover, the prescribed transient and steady-state performance of the closed-loop system can be ensured, meanwhile the output tracking error is always confined within the prescribed funnel boundary.

ii. Instead of utilizing the linear first-order filters, we employ the nonlinear filters with time-varying integral functions to remove the issue of "explosion of complexity". Besides, a common compensation term was introduced in order to eliminate the effects raised by boundary layer errors. Based on Lyapunov stability theorem, it is proven that the zero-error tracking can be achieved.

iii. The proposed control scheme only contains one adaptive tuning parameter of the compensator term in the nonlinear filters so that the implementation of the adaptive law is much easier than backstepping control and conventional DSC. Furthermore, the computational burden is significantly reduced and the design procedure is greatly simplified due to the DSC scheme.

II. PROBLEM STATEMENT AND PRELIMINARIES

2.1 SYSTEM FORMULATION

Consider an uncertain nonlinear strict feedback system below:

$$\begin{aligned} \dot{x}_i &= q_i f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1}, i = 1,\cdots,n-1, \\ \dot{x}_n &= q_n f_n(\bar{x}_n) + g_n(\bar{x}_n)u, \\ y &= x_1, \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1,\cdots,x_i]^T \in \mathbb{R}^i, i = 1,\cdots,n, \bar{x}_n \in \mathbb{R}^n$ are the system states; $q_i i = 1,\cdots,n$ are unknown constant parameters; $f_i(\cdot):\mathbb{R}^n \to \mathbb{R}$ and $g_i(\cdot):\mathbb{R}^n \to \mathbb{R}, i = 1,\cdots,n$ are known smooth differentiable functions; $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the system input and output signals, respectively.

Assumption 2.1: The functions $g_i(\cdot)$ satisfy $|g_i(\bar{x}_i)| \ge \mathrm{g}_0, i = 1,\cdots,n$ in where g_0 a positive constant.

Assumption 2.2: The desired trajectory $y_d(t)$ is continuous, differentiable, bounded and available.

Assumption 2.3: There are sufficient smooth and integrable positive functions $d_i(t)$ satisfying

$$\begin{aligned} &\left|d_i^{(j)}(t)\right| \le d_{i,j}^*, 1 \le i \le n, 1 \le j \le n-1, \\ &\int_0^t d_i(s)ds \le \bar{d}_i, \forall t \ge 0 \end{aligned} \tag{2}$$

with $d_{i,j}^*$ and $\bar{d}_i$ are positive constants.

Remark 2.1: Assumptions 2.1 and 2.2 are common restrictive conditions in adaptive state feedback DSC schemes, which are adopted in many existing literatures, such as [9, 10], [13], [25].

Remark 2.2: According to Assumption 2.3, the time-varying integral functions $d_i(t), 2 \le i \le n$ in filters are sufficiently smooth bounded functions, which are used in the next stability analysis. Some examples for $d_i(t)$ satisfying the Assumption 2.3 are $\frac{a_1}{a_2 t^l + a_3}$, me^{-lt}, where a_1, a_2, a_3, m, l are arbitrary positive constants. In this paper, we chose $d_i(t) = m_i e^{-l_i t}$.

Lemma 2.1: [25] The following inequalities hold all the time

$$0 \le a \tanh\left(\frac{a}{b}\right) \le |a|, \forall a \in \mathbb{R}, b > 0, \tag{3}$$

$$\frac{c}{c+d} \le 1, c \ge 0, d > 0, \text{or} \, c > 0, d \ge 0. \tag{4}$$

Lemma 2.2: [26] If the functions $h(t), \dot{h}(t)$ are bounded and $\lim_{t \to +\infty} \int_0^t h^2(s) ds \le +\infty, \forall t \ge 0$, then

$$\lim_{t \to +\infty} h(t) = 0. \tag{5}$$

The control objectives are that:

i. Design a state feedback control u such that the system output signal y asymptotically tracks a reference signal y_d; ii. Ensure that the tracking error $e_y(t) = y(t) - y_d(t)$ is always satisfied the prescribed funnel boundary.

2.2 FUNNEL CONTROL

As pointed out in [17, 23], a time-varying gain $r(t)$ is introduced to control systems of class of *S* with relative degree r=1 or 2 The systems *S* is governed by the funnel controller with $u(t)$ the control input

$$u(t) = r\left(F_f(t), y(t), \|e(t)\|\right) \times e(t), \tag{6}$$

by evaluating the vertical distance

$$d_v(t) = F_f(t) - \|e(t)\| \tag{7}$$

between the the funnel boundary function $F_f(t)$ and the Euclidian norm $\|e(t)\|$ of error at real time.

The funnel boundary $F_f(t) = 1/j(t)$ is given by the reciprocal of an arbitrarily chosen bounded, continuous function $j(t) > 0$ for all $t \ge 0$ with $\sup_{t \ge 0} j(t) < +\infty$. The funnel boundary is defined as the set

$$F_f : t \to \left\{ e \in \mathbb{R}^n \,\middle|\, j(t) \times \|e(t)\| < 1 \right\}. \tag{8}$$

The control gain of $r(\cdot)$ is adjusted by

$$r\left(F_f(t), y(t), \|e(t)\|\right) = \frac{y(t)}{F_f(t) - \|e(t)\|} \tag{9}$$

to ensure that the error $e(t)$ evolves inside the funnel $F_f(t)$. $y(t)$ is the scaling factor. One can conclude that as the error $e(t)$ approaches the boundary $F_f(t)$, the control gain $r(\cdot)$ will increase. Conversely, if the error $e(t)$ becomes small, the control gain $r(\cdot)$ will decrease.

According to [28], a proper funnel boundary function to prescribe the performance is defined as

$$F_f(t) = s_0 e^{-bt} + s_\infty, \tag{10}$$

where s_0 is the initial value of $F_f(t)$, and $s_\infty = \lim_{t \to \infty} F_f(t), s_0 \ge s_\infty > 0$, and $|e(0)| < F_f(0) = s_0 + s_\infty$, and b is the convergence speed of exponential function. s_0, s_∞, b are the design parameters.

Then, in this paper, the following funnel error variable S_1 is defined as

$$S_1(t) = \frac{e(t)}{F_f(t) - \|e(t)\|}, \tag{11}$$

where the funnel boundary $F_f(t)$ satisfies the condition in (8). This funnel error variable S_1 will be utilized to ensure the prescribed output performance for system (1).

Remark 2.3: In (11), the funnel error variable S_1 is not necessarily limited by S systems because of the condition in (8), thus, the proposed funnel control can be used in various practice. Compared with PPC [16, 28], furthermore, the potential singularity problem is overcome since the inverse of the transformed error is avoided.

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

In this section, the design procedure of the adaptive DSC based on the funnel variable with rigorously stability analysis will derived.

3.1 ADAPTIVE DSC CONTROLLER DESIGN

Step 1. From (11), define the funnel error surface as

$$S_1(t)=\frac{e_y}{F_f-\|e_y\|}, \qquad (12)$$

whose derivative of S_1 with respect to time is

$$\dot{S}_1=\frac{F_f\dot{e}_y-\dot{F}_f e_y}{\left(F_f-\|e_y\|\right)^2}=F_f r_1\dot{e}_y-\dot{F}_f r_1 e_y, \qquad (13)$$

where $r_1=1/\left(F_f-\|e_y\|\right)^2$.

Choose virtual control law x_{2d} and the adaptive law for $\hat{q}_1$ as follows:

$$x_{2d}=-\frac{1}{g_1}\left(\frac{c_1S_1}{F_f r_1}-\frac{\dot{F}_f e_y}{F_f}+\hat{q}_1 f_1-\dot{y}_d\right), \qquad (14)$$

$$\dot{\hat{q}}_1=g_1 r_1 f_1 S_1 F, \qquad (15)$$

where c_1 and g_1 are positive design parameters and $\hat{q}_1$ is the estimation of q_1.

Consider the following Lyapunov function as

$$V_1=\frac{1}{2}S_1^2+\frac{1}{2g_1}\tilde{q}_1^2, \qquad (16)$$

where $\tilde{q}_1=q_1-\hat{q}_1$.

Considering (13), (14) and (15), the time derivative of V_1 becomes

$$\dot{V}_1=-c_1S_1^2+F_f r_1 g_1\left(x_2-x_{2d}\right)S_1. \qquad (17)$$

To avoid the issue of "explosion of complexity", let x_{2d} pass through an adaptive filter. The dynamics of the filter are expressed as

$$t_2\dot{z}_2=-y_2-\frac{t_2 y_2\hat{M}^2}{y_2\tanh\left(\frac{y_2}{d_2(t)}\right)\hat{M}+d_2(t)}-t_2 r_1 g_1 S_1 F_f,$$

$$z_2(0)=x_{2d}(0), \qquad (18)$$

where $y_2:=z_2-x_{2d}$ is the first boundary layer error, $\hat{M}$ denotes the estimate of M that will be presented later; $d_2(t)$ is defined in Assumption 2.3; t_2 is filter time constant.

Step 2: Define the second surface error

$$S_2=x_2-z_2, \qquad (19)$$

differentiating (19) yields

$$\dot{S}_2=q_2f_2+g_2x_3+\frac{y_2}{t_2}+\frac{y_2\hat{M}^2}{y_2\tanh\left(\frac{y_2}{d_2(t)}\right)\hat{M}+d_2(t)}+r_1g_1S_1F_f. \qquad (20)$$

Choose virtual control law x_{3d} and the adaptive law for $\hat{q}_2$ as follows:

$$x_{3d} = -\frac{1}{g_2}\left(c_2 S_2 + \hat{q}_2 f_2 + \frac{y_2}{t_2} + \frac{y_2 \hat{M}^2}{y_2 \tanh\left(\frac{y_2}{d_2(t)}\right)\hat{M} + d_2(t)} + 2r_1 g_1 S_1 F\right), \quad (21)$$

$$\dot{\hat{q}}_2 = g_2 S_2 f_2, \quad (22)$$

where c_2 and g_2 are positive design parameters and $\hat{q}_2$ is the estimation of q_2.

Consider the following Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}S_2^2 + \frac{1}{2g_2}\tilde{q}_2^2, \quad (23)$$

where $\tilde{q}_2 = q_2 - \hat{q}_2$.

Considering (20), (21) and (22), the time derivative of V_2 becomes

$$\dot{V}_2 = -c_1 S_1^2 - c_2 S_2^2 + r_1 g_1 S_1 y_2 F_f + g_2 S_2 \left(x_3 - x_{3d}\right). \quad (24)$$

Let x_{3d} pass through the following adaptive filter

$$t_3 \dot{z}_3 = -y_3 - \frac{t_3 y_3 \hat{M}^2}{y_3 \tanh\left(\frac{y_3}{d_3(t)}\right)\hat{M} + d_3(t)} - t_3 g_2 S_2,$$

$$z_3(0) = x_{3d}(0), \quad (25)$$

where $y_3 := z_3 - x_{3d}$ is the second boundary layer error; $d_3(t)$ is defined in Assumption 2.3; t_3 is filter time constant.

Step i $(3 \le i \le n-1)$: Similar to Step 2, we define the *i*th surface error

$$S_i = x_i - z_i, \quad (26)$$

then

$$\dot{S}_i = q_i f_i + g_i x_{i+1} + \frac{y_i}{t_i} + \frac{y_i \hat{M}^2}{y_i \tanh\left(\frac{y_i}{d_i(t)}\right)\hat{M} + d_i(t)} + g_{i-1} S_{i-1}. \quad (27)$$

Choose virtual control law x_{i+1d} and the adaptive law for $\hat{q}_i$ as follows:

$$x_{i+1d} = -\frac{1}{g_i}\left(c_i S_i + \hat{q}_i f_i + \frac{y_i}{t_i} + \frac{y_i \hat{M}^2}{y_i \tanh\left(\frac{y_i}{d_i(t)}\right)\hat{M} + d_i(t)} + 2g_{i-1} S_{i-1}\right), \quad (28)$$

$$\dot{\hat{q}}_i = g_i S_i f_i, \quad (29)$$

where c_i and g_i are positive design parameters and $\hat{q}_i$ is the estimation of q_i.

Consider the following Lyapunov function as

$$V_i = V_{i-1} + \frac{1}{2}S_i^2 + \frac{1}{2g_i}\tilde{q}_i^2, \quad (30)$$

where $\tilde{q}_i = q_i - \hat{q}_i$.

Considering (27), (28) and (29), the time derivative of V_i becomes

$$\dot{V}_i = -\sum_{k=1}^{i} c_k S_k^2 + r_1 g_1 S_1 y_2 F_f + \sum_{k=2}^{i-1} g_k S_k y_{k+1} + g_i S_i \left(x_{i+1} - x_{i+1d}\right). \quad (31)$$

Let x_{i+1d} pass through the following adaptive filter

$$t_{i+1}\dot{z}_{i+1}=-y_{i+1}-\frac{t_{i+1}y_{i+1}\hat{M}^2}{y_{i+1}\tanh\left(\frac{y_{i+1}}{d_{i+1}(t)}\right)\hat{M}+d_{i+1}(t)}-t_{i+1}g_iS_i,$$

$$z_{i+1}(0)=x_{i+1d}(0), \tag{32}$$

where $y_{i+1}:=z_{i+1}-x_{i+1d}$ is the second boundary layer error; $d_{i+1}(t)$ is defined in Assumption 2.3; t_{i+1} is filter time constant.

Step n: Define the *n*th surface error

$$S_n=x_n-z_n, \tag{33}$$

we have

$$\dot{S}_n=q_nf_n+g_nu+\frac{y_n}{t_n}+\frac{y_n\hat{M}^2}{y_n\tanh\left(\frac{y_n}{d_n(t)}\right)\hat{M}+d_n(t)}+g_{n-1}S_{n-1}. \tag{34}$$

Choose actual control law u and the adaptive law for $\hat{q}_n$ as follows:

$$u=-\frac{1}{g_n}\left(c_nS_n+\hat{q}_nf_n+\frac{y_n}{t_n}+\frac{y_n\hat{M}^2}{y_n\tanh\left(\frac{y_n}{d_n(t)}\right)\hat{M}+d_n(t)}+2g_{n-1}S_{n-1}\right), \tag{35}$$

$$\dot{\hat{q}}_n=g_nS_nf_n, \tag{36}$$

where c_n and g_n are positive design parameters and $\hat{q}_n$ is the estimation of q_n.

Consider the following Lyapunov function as

$$V_n=V_{n-1}+\frac{1}{2}S_n^2+\frac{1}{2g_n}\tilde{q}_n^2, \tag{37}$$

where $\tilde{q}_n=q_n-\hat{q}_n$.

Considering (34), (35) and (36), the time derivative of V_n becomes

$$\dot{V}_n=-\sum_{k=1}^{n}c_kS_k^2+r_1g_1S_1y_2F_f+\sum_{k=2}^{n-1}g_kS_ky_{k+1}. \tag{38}$$

Remark 3.1: The terms $-\frac{t_iy_i\hat{M}^2}{y_i\tanh(y_i/d_i(t))\hat{M}+d_i(t)}$ of the nonlinear filters are designed to compensate the effect of the boundary layer errors y_i. Furthermore, the issue of "explosion of complexity" can also be avoided.

3.2 SATBILITY ANALYSIS

Here, the asymptotic stability and the convergence of the closed-loop system is proven based on Lyapunov theory.

By applying (14), (21), (28) and (35), the time derivatives of all surface errors can be obtained as

$$\dot{S}_1=-c_1S_1+\tilde{q}_1r_1f_1F_f+r_1g_1S_2F_f+r_1g_1y_2F_f,$$

$$\dot{S}_2=-c_2S_2-r_1g_1S_1+\tilde{q}_2f_2+g_2S_3+g_2y_3,$$

$$\dot{S}_i=-c_iS_i-g_{i-1}S_{i-1}+\tilde{q}_if_i+g_iS_{i+1}+g_iy_{i+1},\mathrm{i}=3,\cdots,\mathrm{n}-1$$

$$\dot{S}_n=-c_nS_n-g_{n-1}S_{n-1}+\tilde{q}_nf_n. \tag{39}$$

Considering (18), (25) and (32), the derivatives of the boundary layer errors $y_{i+1}=z_{i+1}-x_{i+1d}$ are

$$\dot{y}_2 = -\frac{y_2}{t_2} - \frac{y_2 \hat{M}^2}{y_2 \tanh\left(\frac{y_2}{d_2(t)}\right)\hat{M} + d_2(t)} - r_1 g_1 S_1 F_f - \frac{\partial x_{2d}}{\partial x_1}\dot{x}_1 - \frac{\partial x_{2d}}{\partial \hat{q}_1}\dot{\hat{q}}_1 - \frac{\partial x_{2d}}{\partial y_d}\dot{y}_d - \frac{\partial x_{2d}}{\partial \dot{y}_d}\ddot{y}_d$$

$$= -\frac{y_2}{t_2} - \frac{y_2 \hat{M}^2}{y_2 \tanh\left(\frac{y_2}{d_2(t)}\right)\hat{M} + d_2(t)} - r_1 g_1 S_1 F_f + B_2(\cdot),$$

$$\dot{y}_{i+1} = -\frac{y_{i+1}}{t_{i+1}} - \frac{y_{i+1} \hat{M}^2}{y_{i+1} \tanh\left(\frac{y_{i+1}}{d_{i+1}(t)}\right)\hat{M} + d_{i+1}(t)} - g_i S_i - \sum_{k=1}^{i}\frac{\partial x_{i+1d}}{\partial x_k}\dot{x}_k - \frac{\partial x_{i+1d}}{\partial \hat{q}_i}\dot{\hat{q}}_i - \frac{\partial x_{i+1d}}{\partial y_i}\dot{y}_i$$

$$-\frac{\partial x_{i+1d}}{\partial \hat{M}}\dot{\hat{M}} - \frac{\partial x_{i+1d}}{\partial y_d}\dot{y}_d - \frac{\partial x_{i+1d}}{\partial \dot{y}_d}\ddot{y}_d - \frac{\partial x_{i+1d}}{\partial d_{i+1}(t)}\dot{d}_{i+1}(t) \tag{40}$$

$$= -\frac{y_{i+1}}{t_{i+1}} - \frac{y_{i+1} \hat{M}^2}{y_{i+1} \tanh\left(\frac{y_{i+1}}{d_{i+1}(t)}\right)\hat{M} + d_{i+1}(t)} - g_i S_i + B_{i+1}(\cdot), \mathrm{i} = 2,\cdots,\mathrm{n}-1$$

where $B_2(\cdot)$ and $B_{i+1}(\cdot)$ are continuous functions.
Now, let the Lyapunov function be defined as

$$V = V_n + \sum_{i=1}^{n-1}\frac{1}{2}y_{i+1}^2 + \frac{1}{2h}\tilde{M}^2 + \sum_{i=1}^{n-1} d_{i+1}(t), \tag{41}$$

where h is positive design parameters.
We have the following theorem:

Theorem : Consider the closed-loop control system (1) with the virtual control laws $\bar{x}_i$ (14), (21) and (28), and the control law $u(t)$ (35), the adaptive laws $\dot{\hat{q}}_i$ (15), (22), (29), (36) and $\dot{\hat{M}}$ (47), subject to Assumption 2.1–2.3. For $V(0) \le C_2$ and $y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le C_1$, where C_1, C_2 are given positive constants, then, if the initial condition of tracking error $e_y(t)$ fulfills $|e_y(0)| \le F_f(0)$, then there exist the following properties:
i. All signals in the closed-loop system are semi-globally bounded.
ii. The output tracking error $e_y(t)$ converges to zero asymptotically, besides it can be guaranteed within in a prescribed funnel boundary.

Proof: Define the following compact sets:

$$\Omega_1 = \left\{[y_d, \dot{y}_d, \ddot{y}_d]^T : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le C_1\right\}, \tag{42}$$

$$\Omega_2 = \left\{V(t) \le C_2\right\}. \tag{43}$$

Note that $\Omega_1 \times \Omega_2$ is also a compact set. Therefore, there exist unknown positive constant M_{i+1} such that $|B_{i+1}(\cdot)| \le M_{i+1}, i = 1,\cdots,\mathrm{n}-1$ on $\Omega_1 \times \Omega_2$. Let $M := \max\{M_2,\cdots,M_n\}$ and it is estimated by $\hat{M}$, then taking the time derivative of (41) yields

$$\dot{V} \le -\sum_{i=1}^{n} c_i S_i^2 - \sum_{i=1}^{n-1}\frac{y_{i+1}^2}{t_{i+1}} + \sum_{i=1}^{n-1} M|y_{i+1}| - \sum_{i=1}^{n-1}\frac{y_{i+1}^2 \hat{M}^2}{y_{i+1}\tanh\left(y_{i+1}/d_{i+1}(t)\right)\hat{M} + d_{i+1}(t)} - \frac{1}{h}\tilde{M}\dot{\hat{M}}. \tag{44}$$

From Lemma 2.1, we have

$$\begin{aligned} &M|y_{i+1}| - \frac{y_{i+1}^2 \hat{M}^2}{y_{i+1}\tanh\left(y_{i+1}/d_{i+1}(t)\right)\hat{M} + d_{i+1}(t)} \\ &\le \frac{|y_{i+1}|\hat{M} d_{i+1}(t)}{|y_{i+1}|\hat{M} + d_{i+1}(t)} + \tilde{M}|y_{i+1}| \le d_{i+1}(t) + \tilde{M}|y_{i+1}|, \end{aligned} \tag{45}$$

then, (44) can be written

$$\dot{V} \le -\sum_{i=1}^{n} c_i S_i^2 - \sum_{i=1}^{n-1} \frac{y_{i+1}^2}{t_{i+1}} + \sum_{i=1}^{n-1} \left(\tilde{M} \left| y_{i+1} \right| - \frac{1}{h} \tilde{M} \dot{\hat{M}} \right), \tag{46}$$

thus, the adaptive laws for $\hat{M}$ is chosen as

$$\dot{\hat{M}} = h \sum_{i=1}^{n-1} \left| y_{i+1} \right|, \mathrm{i} = 1, \cdots, \mathrm{n} - 1, \tag{47}$$

then, we can obtain

$$\dot{V} \le -\sum_{i=1}^{n} c_i S_i^2 - \sum_{i=1}^{n-1} \frac{y_{i+1}^2}{t_{i+1}}. \tag{48}$$

Integrating (48) over [0, t], we have

$$V(t) \le V(0) - \int_0^t \left(\sum_{i=1}^{n} c_i S_i^2(i) + \sum_{i=1}^{n-1} \frac{y_{i+1}^2(i)}{t_{i+1}} \right) di \le V(0), \tag{49}$$

which implies that $S_1, \cdots, S_n, \hat{q}_1, \cdots, \hat{q}_n, y_2, \cdots, y_n, \hat{M}$ are bounded. Consequently, we can deduce that $x_1, \cdots, x_n, x_{2d}, \cdots, x_{nd}$ and u are bounded. Moreover, from (49), we have

$$\int_0^t \sum_{i=1}^{n} c_i S_i^2(i) di \le V(0). \tag{50}$$

From Lemma 2.2, it is concluded that

$$\lim_{t \to \infty} S_1 = 0, \tag{51}$$

which implies that the asymptotic tracking is achieved due to (12).
From (49) and (12), we have

$$\left(e_y / \left(F_f - \left\| e_y \right\| \right) \right)^2 \le 2V(0), \tag{52}$$

if $0 < V(0) \le 1/2$, then

$$2V(0) \left(F_f^2 - 2F_f \left\| e_y \right\| \right) \ge 0, \tag{53}$$

which means that

$$\left(F_f^2 - 2F_f \left\| e_y \right\| \right) \ge 0, \tag{54}$$

thus, we can obtain

$$\left\| e_y \right\| \le 1/2 \left| F_f \right| \le \left| F_f \right|. \tag{55}$$

Therefore, all signals of the closed-loop systems are semi-globally bounded and the tracking error is always well-kept within the prescribed funnel boundary. This completes the proof. □

Remark 3.2: It is noted that our designed control strategy can guarantee both prescribed tracking performance and state steady. Moreover, the computational load is greatly reduced due to the DSC technique, which makes the design process and the controller quite simple and practical.

IV. SIMULATION RESULTSEXAMPLES

In this section, two practical simulations are conducted to verify the effectiveness and the high performance of the proposed control design.

Example 1. A single-link robot manipulator model [29] is considered whose motion dynamics expression can be given by

$$\begin{cases} M_q \ddot{q} + 1/2 mgl \sin q = t_u, \\ y = q, \end{cases} \tag{57}$$

where $q, \dot{q}, \ddot{q}$ are the angular position, the velocity and acceleration, M_q is the inertia, m is the link mass, $g = 9.8\mathrm{m/s}^2$ is the gravity, l is the length of the link, and t_u is the control force.

Define $x_1 = q, x_2 = \dot{q}, u = t_u$, then (57) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2, x_1(0) = 0.2, \\ \dot{x}_2 = -0.5mgl\sin x_1 / M_q + u / M_q, x_2(0) = 0, \\ y = x_1, \end{cases} \tag{58}$$

which implies that $f_1(x_1) = 0, g_1(x_1) = 1, f_2(x_1, x_2) = -0.5mgl\sin x_1 / M_q, g_2(x_1, x_2) = 1/M_q$.

In this simulation, we choose $M_q = 0.5\text{kg/m}^2, m = 1\text{kg}, l = 1\text{m}$. The reference signal is given by $y_d = 0.1\sin t - 0.1\cos 2t$. The selected funnel boundary function is $F_f(t) = 0.3e^{-2t} + 0.01$. Based on the initialization technique, the design parameters and the initial conditions of adaptive mechanisms are set as $c_1 = 10, c_2 = 4, g_1 = 1, g_2 = 3, h = 2, t_2 = 0.01, d_2(t) = 50e^{-0.1t}$. By applying the DSC method and the proposed AFC strategy, simulation results are shown in Figures 1–3, from which it can be observed that our proposed control method has smaller overshoots and gains better control performance.

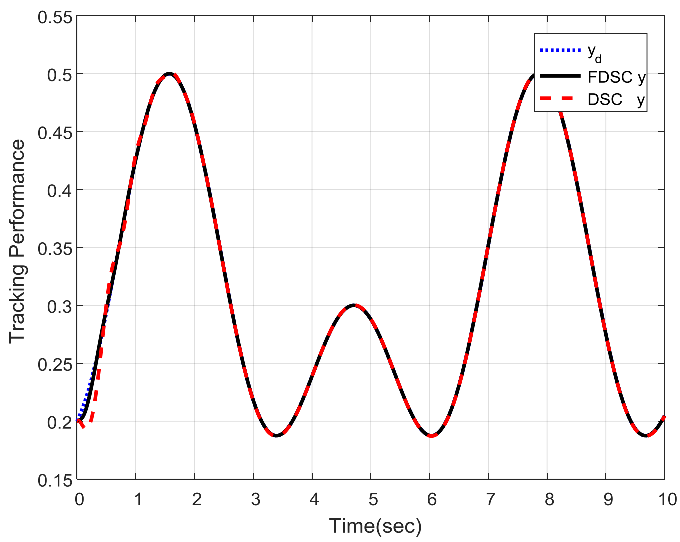

Figure1: Tracking performance for Example 1.

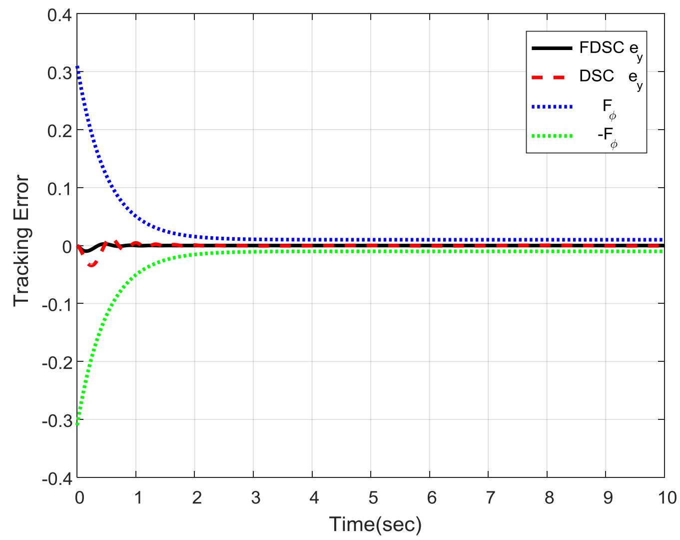

Figure2: Tracking error for Example 1.

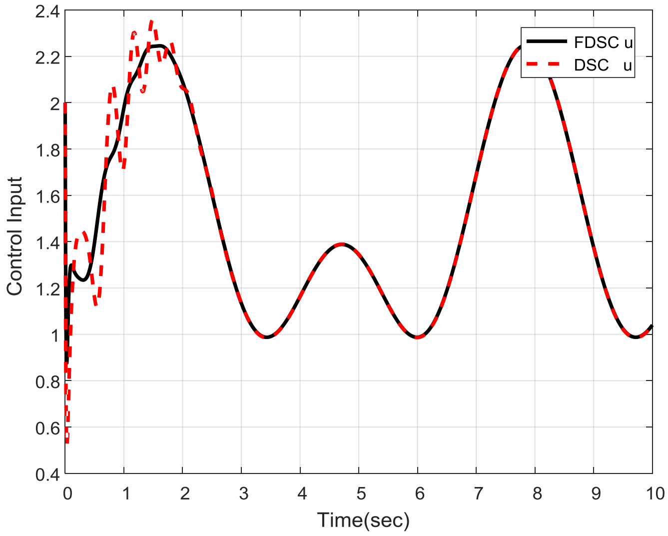

Figure3: Control input for Example 1.

Example 2. To further show the high performance of the proposed algorithm, an electromechanical dynamic system [30] is expressed as

$$\begin{cases} D\ddot{q} + B_m\dot{q} + N\sin q = t_m, \\ M_e\dot{t}_m + Ht_m = V_m - K_M\dot{q}. \end{cases} \quad (59)$$

Define $x_1 = q, x_2 = \dot{q}, x_3 = t_m, u = V_m$, then (59) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -\left(-N\sin x_1 - B_m x_2\right)/D + x_3/D, \\ \dot{x}_3 = \left(-K_m x_2 - Hx_3\right)/M_e + u/M_e, \\ y = x_1, \end{cases} \quad (60)$$

which implies that $f_1(x_1) = 0, g_1(x_1) = 1, f_2(\bar{x}_2) = -N\sin x_1 - B_m x_2, g_2(\bar{x}_2) = 1/D, f_3(\bar{x}_3) = -K_m x_2 - Hx_3, .$ $g_3(\bar{x}_3) = 1/M_e$. The parameters are set as $D = 1, M_e = 0.05, B_m = 1, K_m = 10, H = 0.5, N = 10$.

Then, the initial values of the system (60) are chosen as $x(0) = [0.2, 0.1, 0]^T$. The reference signal is $y_d = 0.1\sin t, y_d(0) = 0.2$. The funnel boundary function $F_f(t)$ is set the same as Example 1. Similar as Example 1, by introducing the initialization technique, the design parameters and the initial conditions of adaptive mechanisms are set as $c_1 = 25, c_2 = 15, c_3 = 8, g_1 = 1, g_2 = 5, g_3 = 3, h = 2, t_2 = t_3 = 0.01, d_2(t) = 50e^{-0.6t}, d_3(t) = 100e^{-0.1t}$. By applying the DSC method and the proposed AFC strategy, simulation results are shown in Figures 4–6, from which it is clear that the control performance has been improved significantly and prescribed tracking performance has been achieved.

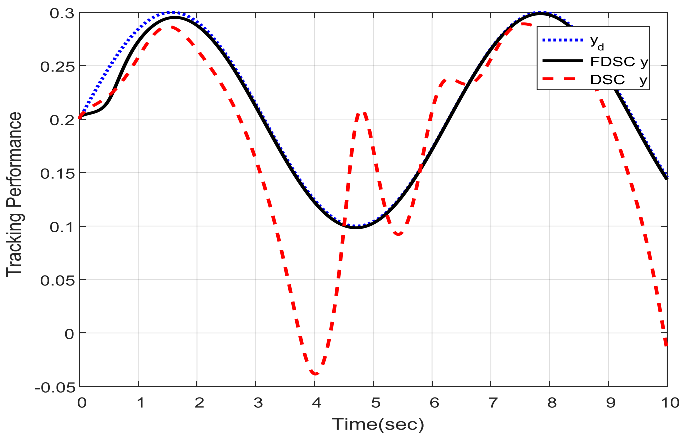

Figure4: Tracking performance for Example 2.

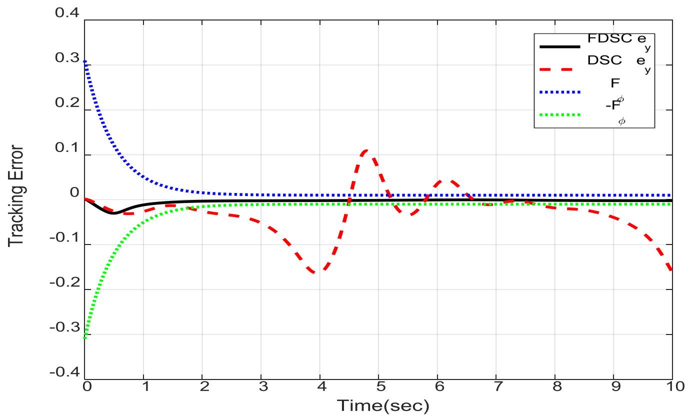

Figure5: Tracking error for Example 2.

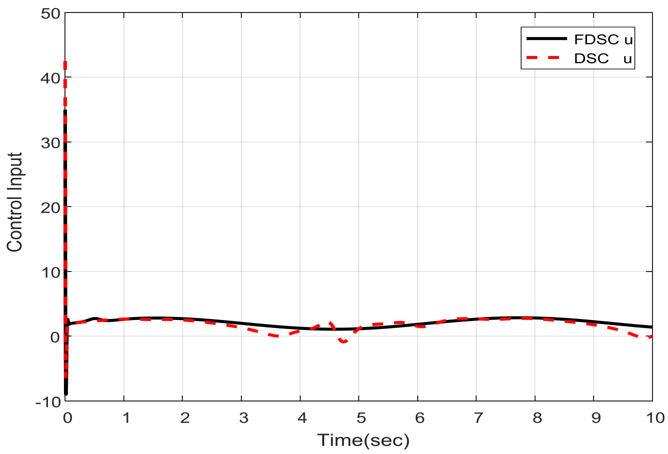

Figure6: Control input for Example 2.

V. CONCLUSION

In this paper, an improved adaptive funnel DSC method has been proposed for uncertain strict feedback systems. Incorporating the funnel boundary function into DSC technique, we have shown that the new control design can guarantee both the prescribed transient boundary and steady-state performance. Moreover, it has been proved that the controller design procedure and the computational costs can be significantly alleviated due to DSC. On the other hand, by introducing the compensation terms of boundary layer errors in each step of filters, the favorable asymptotic convergence can be achieved. The proposed control methodology of this paper is validated through simulation results.

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